

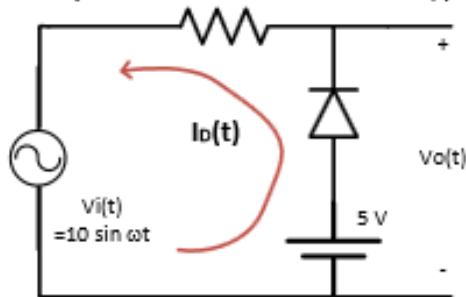
ENEE2103- Circuits and Electronics lab

Electronics Theoretical Background Summary

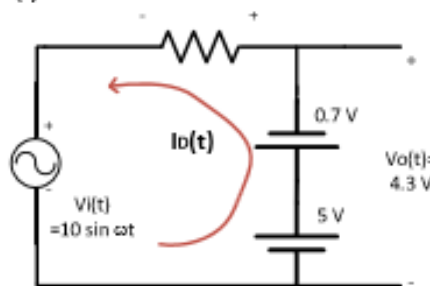
1. Diode Applications

⇒ Clipper-Limiter

Example: Calculate and sketch $V_o(t)$ using simplified diode model



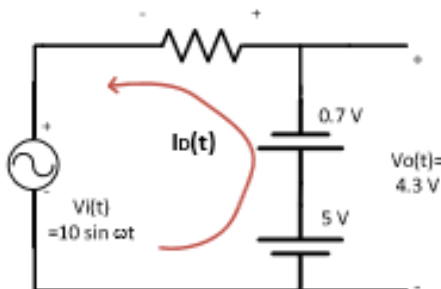
1) Assume diode is ON, so we replace it by 0.7 V and $i_D(t)$ must be > 0



$$5V - 0.7V - i_D(t) \cdot R - V_i(t) = 0$$

$$i_D(t) \cdot R = 4.3V - V_i(t)$$

$$i_D(t) = \frac{4.3V - V_i(t)}{R} > 0$$

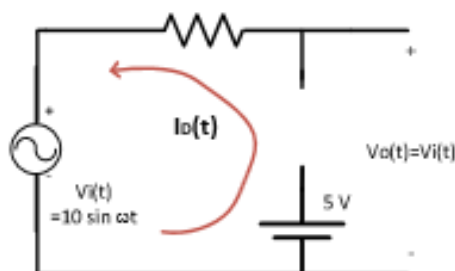


$$\therefore 4.3V - V_i(t) > 0$$

$$\Rightarrow V_i(t) < 4.3V$$

when $V_i(t) < 4.3V$ diode is ON and $V_o(t) = 4.3V$

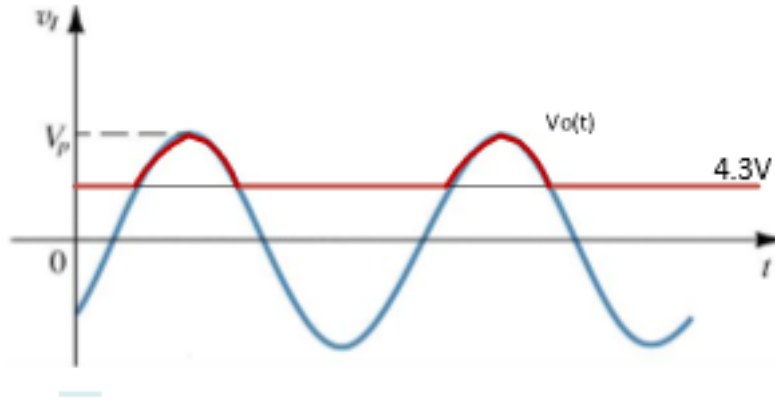
2) Otherwise, When $V_i(t) > 4.3V$, Diode will be off and it is replaced by open circuit



$$\Rightarrow V_i(t) > 4.3V$$

$$V_o(t) = V_i(t)$$

when $V_i(t) < 4.3V$, diode is ON & $V_o(t) = 4.3V$
 when $V_i(t) > 4.3V$, diode is off & $V_o(t) = V_i(t)$

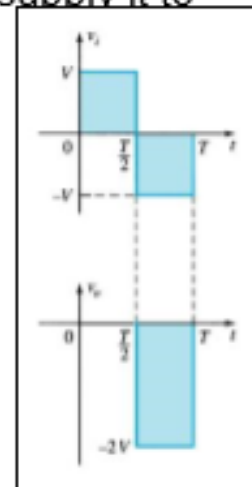
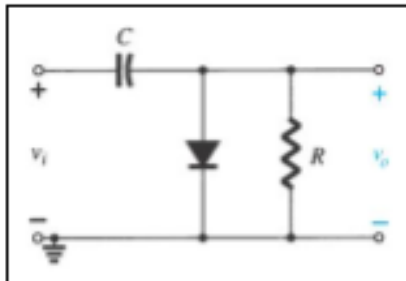


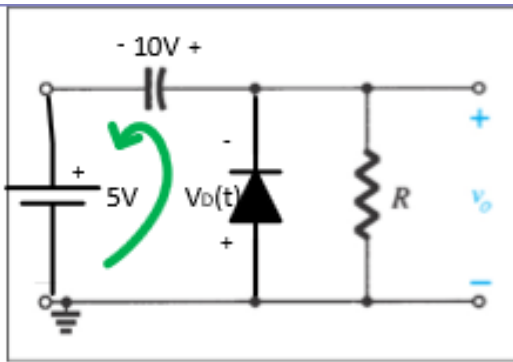
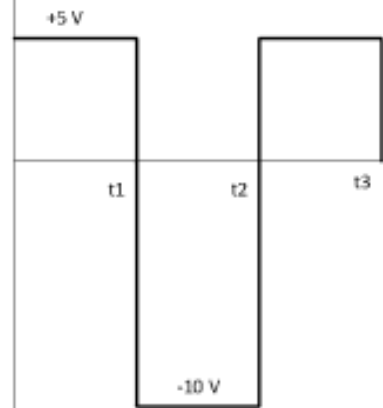
⇒ Diode Clamper

Clampers

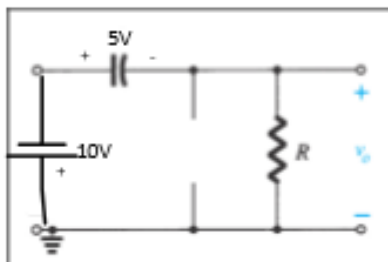
Function: A Clamper shifts the input waveform up or down (adds a dc offset) while keeping its shape and peak to peak value unchanged.

It consists of a diode and capacitor (and maybe a series dc source) that can be combined to “clamp” an AC signal to a specific DC level and supply it to the load R



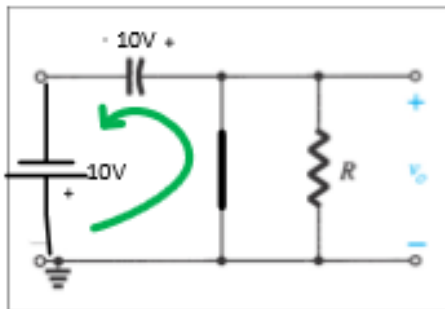
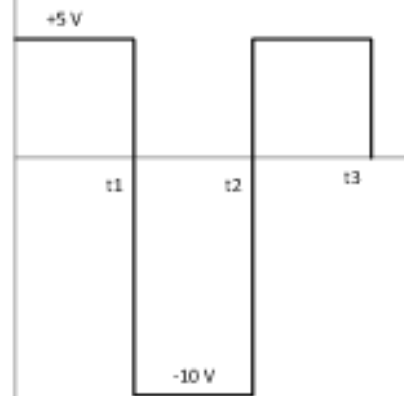

 $v_i(t)$


KVL around the loop: $10 + 5 + V_D(t) = 0$
 $\Rightarrow V_D(t) = -15 \text{ V} < 0$, \therefore diode is OFF



$$V_o(t) = -V_D(t) = 15 \text{ V}$$

Cap is charged to 10V with shown polarity due to diode forward current $V_o(t) = 0 \text{ V}$


 $v_i(t)$


2) for $t_2 < t < t_3$ voltage source reverses polarity, $V_i(t) = +5\text{V}$ while Cap keeps its charge $V_c = 10\text{V}$

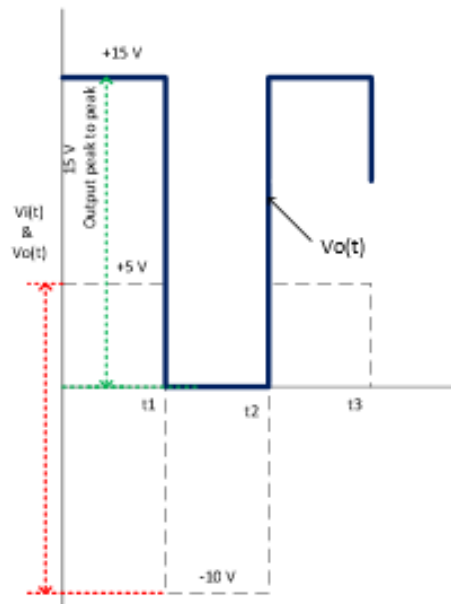
Afterwards for any value of the given $V_i(t)$ diode remains OFF and $V_o(t) = V_i(t) + 10$

∴ the clamper charges a cap and uses this charge to add up to the input to shift it up or down (i.e. add dc offset)

Important Note

For Proper Clamping action, $\tau_{discharge}$ must be large enough (at least 10 times the period of the input waveform)

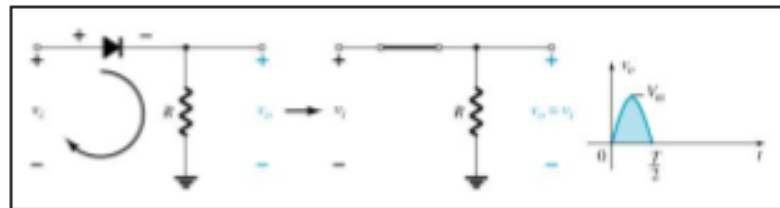
$$\tau_{discharge} = R.C > 10(t_1 + t_2)$$



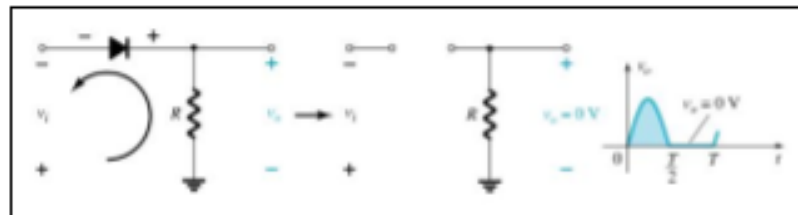
⇒ Diode Rectifier

Half Wave Rectifier

1) When $V_i(t) > 0$,
Diode is ON
 $V_o = V_i$



2) When $V_i(t) < 0$,
Diode is OFF
 $V_o = 0$



The diode conducts only when it is forward biased, therefore only half of the AC cycle passes through the diode to the output.

$$V_{AVG} = \frac{1}{T} \int_0^T V_m \sin(\omega t) dt$$

Half Wave Rectifier

$$\begin{aligned}
 V_{\text{AVG}} &= \frac{1}{T} \int_0^T V_m \sin(\omega t) dt \\
 &= \frac{1}{2\pi} \int_0^{\pi} V_m \sin(\theta) d\theta \\
 &= \frac{1}{2\pi} [-V_m \cos(\theta)]_0^{\pi} = \frac{V_m}{2\pi} [-\cos(\pi) - (-\cos(0))]
 \end{aligned}$$

$$\begin{aligned}
 V_{\text{AVG}} &= \frac{V_m}{2\pi} [-(-1) - (-1)] \\
 &= \frac{2V_m}{2\pi} = \frac{V_m}{\pi} \cong 0.318V_m
 \end{aligned}$$

$$T = T_o \text{ and } f = f_o$$

(period and frequency of the rectified waveform is the same as ac input)

Bridge Full-Wave Rectifier

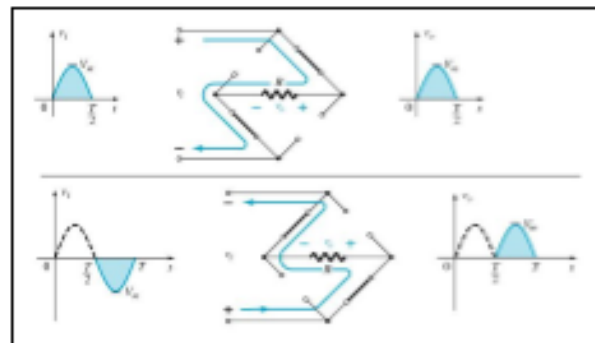
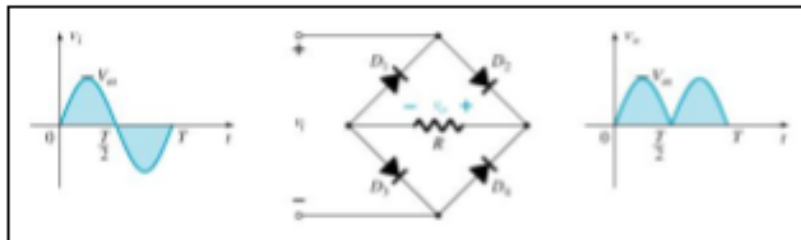
A full-wave rectifier with four diodes that are connected in a bridge configuration

1) When $V_i(t) > 0$,
 D2, D3 are ON
 D1, D4 are OFF

$$\Rightarrow V_o(t) = V_i(t)$$

2) When $V_i(t) < 0$,
 D2, D3 are OFF
 D1, D4 are ON

$$\Rightarrow V_o(t) = -V_i(t)$$



$$V_{AVG} = \frac{2V_m}{\pi} \cong 0.636V_m$$

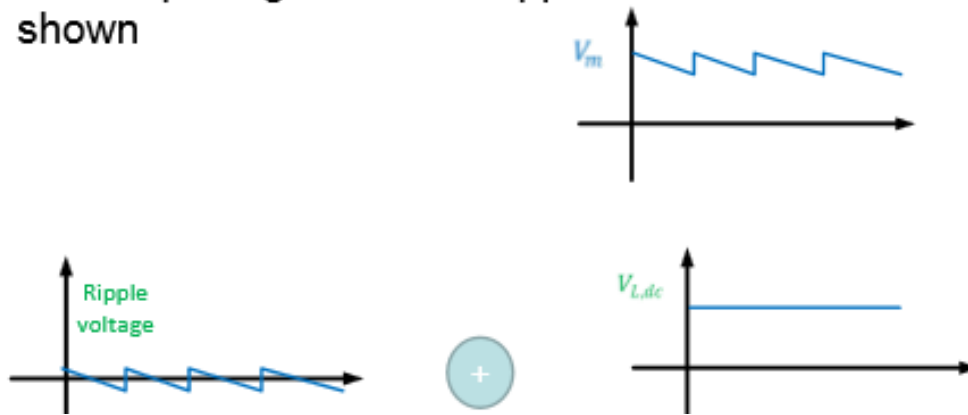
$$T = \frac{T_o}{2} \text{ and } f = 2f_o$$

(period and frequency of the rectified waveform is not the same as ac input)

- Ripple factor is an indicator for the effectiveness of the filter

$$r = \frac{\text{RMS(ripple voltage)}}{\text{Average value of the output signal}} \times 100\%$$

- The output signal can be approximated as shown



- Also for a triangular signal,

$$\text{RMS Value} = \frac{\text{Peak Value}}{\sqrt{3}}$$

OR

$$\text{RMS Value} = \frac{\text{Peak to peak Value}}{2\sqrt{3}} = \frac{V_{Lr,p-p}}{2\sqrt{3}}$$

$$V_{L,dc} = V_m - \frac{1}{2} V_{Lr,p-p}$$

a) For Half Wave Rectifier

$$t_2 - t_1 \cong T_o = \frac{1}{f_o}$$

$$V_{Lr,p-p} = V_m \left(\frac{T_o}{RC} \right) = V_m \left(\frac{1}{f_o RC} \right)$$

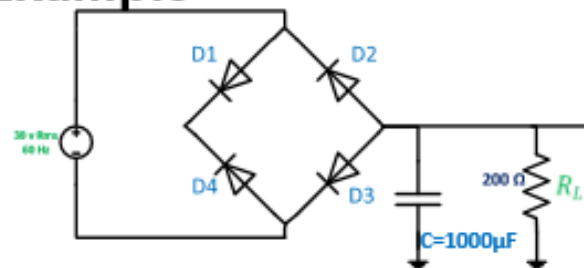
b) For Full Wave Rectifier

$$t_2 - t_1 \cong \frac{1}{2} T_o = \frac{1}{2f_o}$$

$$V_{Lr,p-p} = V_m \left(\frac{T_o}{2RC} \right) = V_m \left(\frac{1}{2f_o RC} \right)$$

► Find the ripple factor r

Example



- $V_{L,dc} = V_m - \frac{1}{2} \frac{V_m}{2f_o R_L C} = 41.54 \text{ v}$

$$V_{Lr,p-p} = \frac{V_m}{2f_o R_L C} = 1.7677 \text{ v}$$

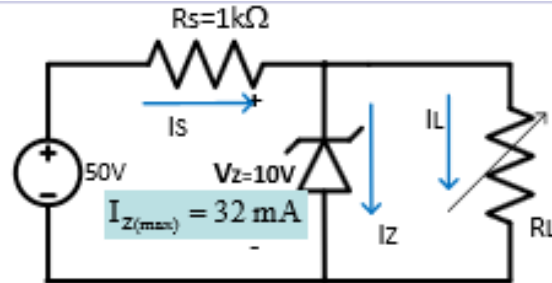
- RMS (ripple voltage) = $\frac{V_{Lr,p-p}}{2\sqrt{3}} = 0.51 \text{ v rms}$

$$\therefore r = \frac{0.51}{41.54} \times 100\%$$

$$r = 1.2277 \%$$

⇒ **Zener Diode VR****Example**

- 1) Determine Range of R_L & I_L that will result in V_L being maintained at 10V
- 2) Determine the power rating of the zener diode

**SOLUTION**

- 1) To find $R_{L(min)}$ that will turn the zener diode ON :

$$V_L = V_Z = \frac{R_L}{R_L + R_S} V_i \Rightarrow$$

$$R_L = \frac{R_S}{V_i - V_Z} V_Z$$

$$R_{L(min)} = \frac{1 \text{ k}\Omega}{50 - 10} 10 = 250 \Omega$$

$$250 \Omega \leq R_L \leq 1.25 \text{ k}\Omega$$

- 2) To find $R_{L(max)} \Rightarrow$ we need $I_{L(min)}$

$$I_{L(min)} = I_S - I_{Z(max)}$$

$$I_S = \frac{V_i - V_Z}{R_S} = \frac{50 - 10}{1 \text{ k}\Omega} = 40 \text{ mA}$$

$$I_{L(min)} = 40 - 32 = 8 \text{ mA}$$

$$\Downarrow$$

$$R_{L(max)} = \frac{V_L}{I_{L(min)}} = \frac{10 \text{ V}}{8 \text{ mA}} = 1.25 \text{ k}\Omega$$

- 3) $P_{Z(max)} = V_Z \cdot I_{Z(max)} = 10 \text{ V} \cdot 32 \text{ mA} = 320 \text{ mW}$

The zener diode is chosen with power rating $\geq P_{Z(max)}$

Example

Find the range of values of V_i that will maintain the zener in the ON (regulation) State

$$V_{in(min)} = \frac{R_L + R_S}{R_L} V_Z$$

$$= \frac{1200 + 220}{1200} 20 = 23.07 \text{ V}$$

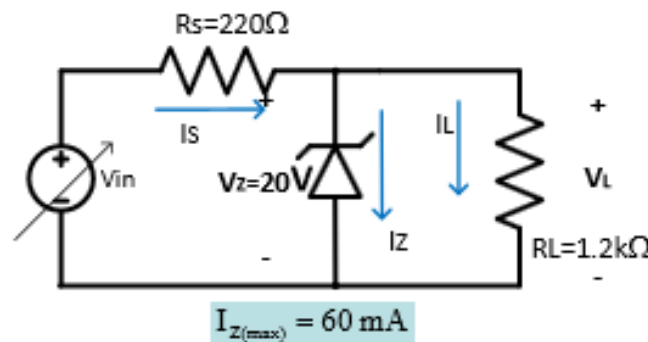
$$I_S = I_Z + I_L$$

$$I_{S(max)} = I_{Z(max)} + I_L$$

$$I_L = \frac{V_L}{R_L} = \frac{20 \text{ V}}{1200 \Omega} = 16.67 \text{ mA}$$

$$I_{S(max)} = 60 \text{ mA} + 16.67 \text{ mA} = 76.67 \text{ mA}$$

$$V_{i(max)} = (76.67 \text{ mA}) \cdot (220 \Omega) + 20 \text{ V} = 36.87 \text{ V}$$



$$\therefore 23.07 \text{ V} \leq V_{in} \leq 36.87 \text{ V}$$

2. BJT dc analysis

Transistor biasing:

- ✓ In order to operate properly as an amplifier, it's necessary to correctly bias the two pn-junctions with external voltages.
- ✓ Depending upon external bias voltage polarities used; the transistor works in one of **four regions** (modes). npn transistor modes of operation
- ✓ For transistor to be used as an Active device (**Amplifier**); the **emitter-base junction** must be **forward** bias, while the **collector-base junction** must be **reverse** biased.

Junction/ Mode	BE	BC	Remarks
Saturation Mode	Forward	Forward	Equivalent to short circuit $I_C = I_C(\text{sat})$ $V_{CE} = V_{CE}(\text{sat}) \approx -0.2V$
Active Mode (Linear Region)	Forward	Reverse	I_C proportional to I_B V_{CE} defined by circuit
Cut-off Mode	Reverse	Reverse	Equivalent to open circuit $I_C = I_B = 0$ V_{CE} defined by circuit
Inverse Mode	Reverse	Forward	Rarely used and will not be discussed in this course

Basic BJT Amplifiers Circuits

BJT in Active Mode

$$I_E = I_B + I_C$$

$$\alpha \approx \frac{I_C}{I_E}$$

$$\beta \approx \frac{I_C}{I_B} \quad \text{---common-emitter current gain}$$

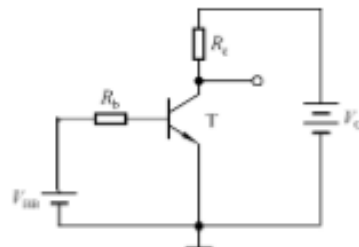
$$\beta = \frac{\alpha}{1-\alpha}$$

BJT DC Analysis

- Make sure the BJT current equations and region of operation match

$$V_{BE} > 0, \\ V_{BC} < 0, \rightarrow V_E < V_B < V_C$$

- Utilize the relationships (β and α) between collector, base, and emitter currents to solve for all currents



$$\begin{cases} I_E = I_C + I_B = (1 + \beta)I_B \\ I_C = \beta I_B \\ I_C = \alpha I_E \end{cases}$$

1. In the **cutoff** region :

$$I_B = I_C = I_E = 0$$

2. In the **active** region :

$$I_C = \alpha I_E$$

$$I_C = \beta I_B$$

$$I_E = (\beta + 1)I_B$$

$$V_{BE} = 0.7 \text{ v} \quad , \quad \text{Si} \quad , \quad \text{nnp}$$

$$V_{BE} = -0.7 \text{ v} \quad , \quad \text{Si} \quad , \quad \text{pnp}$$

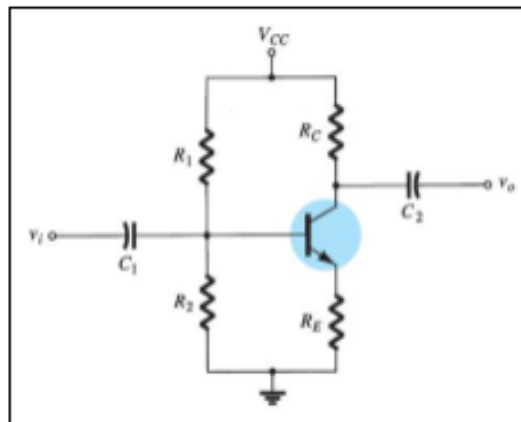
$$V_{CE} > V_{CE,sat} = 0.2 \text{ v} \quad , \quad \text{Si} \quad , \quad \text{nnp}$$

$$V_{CE} < V_{CE,sat} = -0.2 \text{ v} \quad , \quad \text{Si} \quad , \quad \text{pnp}$$

4) Voltage Divider Bias

This is a very stable bias circuit.

The currents and voltages are nearly independent of any variations in β if the circuit is designed properly



Exact Analysis

We must try to make I_B as close as possible to zero

Thevenin Equivalent circuit for the circuit left of the base is done

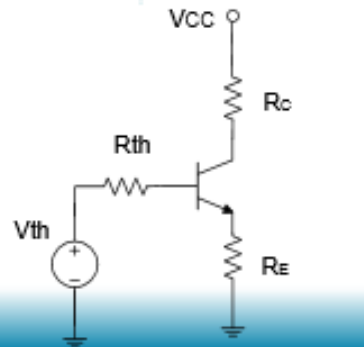
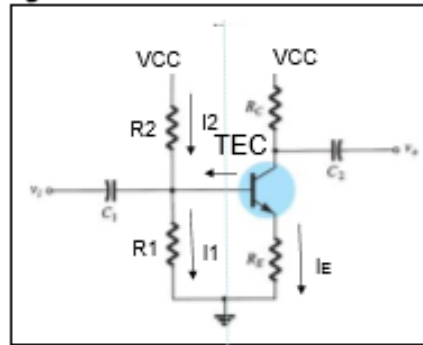
$$V_{th} = \frac{R_2 V_{CC}}{R_1 + R_2}$$

$$R_{th} = R_1 // R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

$$V_{th} = I_B R_{th} + V_{BE} + I_E R_E$$

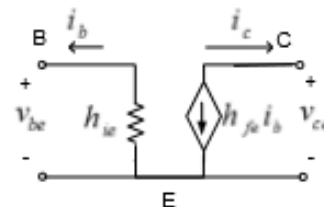
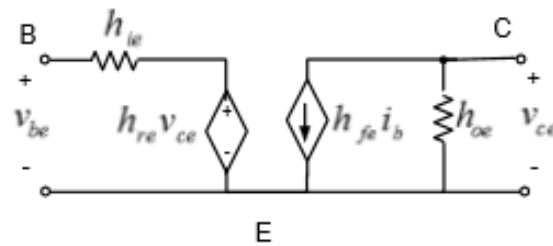
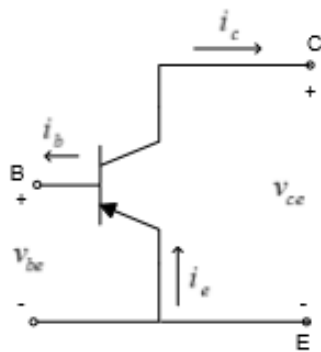
$$\text{but } I_B = \frac{I_E}{\beta + 1}$$

$$\therefore I_{E(\text{exact})} = \frac{V_{th} - V_{BE}}{\frac{R_{th}}{\beta + 1} + R_E}$$



3. BJT amplifier ac analysis (Common Emitter, Common Collector, Common Base)

Common Emitter and Common Collector Configuration



Value of h_{ie}

Base Emitter is a pn junction similar to a diode
 h_{ie} is the dynamic resistance of the pn junction

In a diode:

$$r_d = \frac{V_T}{I_{DQ}} \Rightarrow$$

$$h_{ie} = \frac{V_T}{I_{BQ}} = \frac{V_T}{\frac{I_{CQ}}{h_{fe}}} = \frac{h_{fe} V_T}{I_{CQ}}$$

I_{BQ} dc value of base current

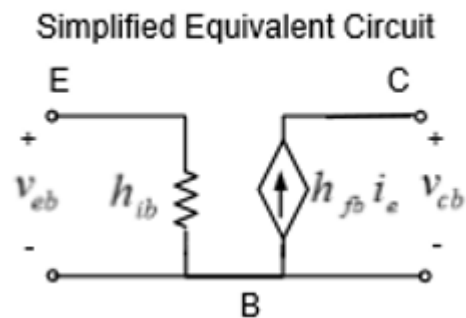
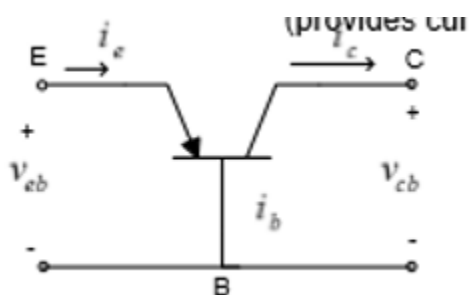
I_{CQ} dc value of collector current

$$h_{fe} = \beta$$

$$V_T = 25.69 \text{ mV @ } 25^\circ\text{C}$$

Common-Base Configuration

(provides current gain and some voltage gain)



$$h_{ib} = \frac{V_T}{I_{EQ}}$$

$$h_{fb} = \alpha$$

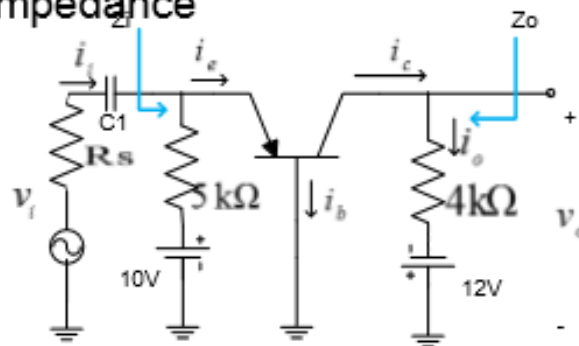
$$V_T = 25.69 \text{ mV @ } 25^\circ\text{C}$$

$$h_{ie} > h_{ib}$$

BJT Amplifier Analysis

When Analyzing Amplifier Circuits, we usually want to find some or all of the following quantities:

- 1) $A_v = V_o/V_i$, small signal voltage gain
- 2) $A_i = i_o/i_i$, small signal current gain
- 3) Z_i Input Impedance
- 4) Z_o Output Impedance



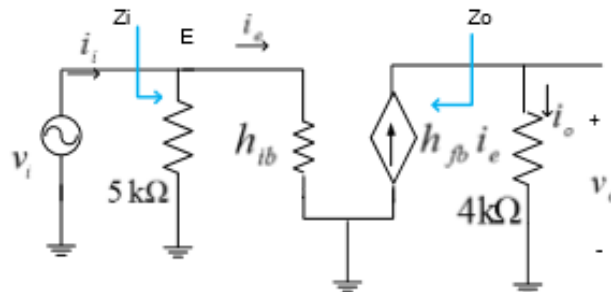
BJT Amplifier Analysis

Solution: (with $R_s=0$)

We draw the ac small signal equivalent circuit

Capacitors \Rightarrow replaced by short circuit

DC sources are killed ,



$$h_{ib} = \frac{V_T}{I_{EQ}}$$

$$h_{fb} = \alpha \cong 1$$

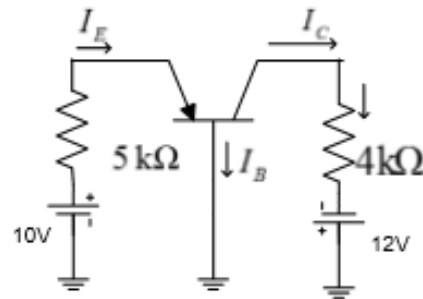
I_{EQ} must be calculated from DC analysis

DC Analysis

DC Equivalent Circuit:

-Cap ==> open

-Kill ac sources ==>



$$10 = 5 \text{ k}\Omega \cdot I_{EQ} + V_{EB}$$

$$I_{EQ} = \frac{10 - 0.7}{5 \text{ k}\Omega} = 1.86 \text{ mA}$$

$$h_{ib} = \frac{V_T}{I_{EQ}} = \frac{25.69 \text{ mV}}{1.86 \text{ mA}} = 13.98 \Omega$$

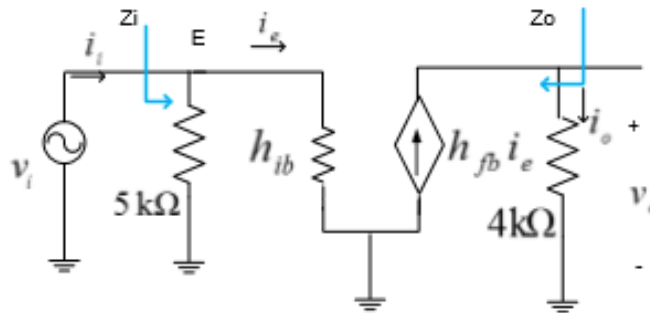
Ac ss equivalent circuit

$$1) A_V = \frac{v_o}{v_i}$$

$$v_o = i_o \cdot 4 \text{ k}\Omega$$

$$i_o = h_{fb} \cdot i_e$$

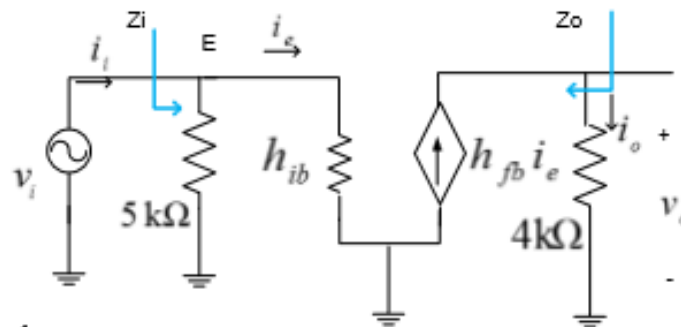
$$i_e = \frac{v_i}{h_{ib}}$$



$$A_V = \frac{v_o}{v_i} = \frac{v_o}{i_o} \cdot \frac{i_o}{i_e} \cdot \frac{i_e}{v_i} \quad \Rightarrow \quad A_V = (4 \text{ k}\Omega) \cdot (h_{fb}) \left(\frac{1}{h_{ib}} \right)$$

$$= (4 \text{ k}\Omega) \cdot (1) \left(\frac{1}{13.98} \right) = 286 > 1$$

Zi & Zo



3) Input Impedance

$$Z_i = (h_{ib} // 5 \text{ k}\Omega) = \left(\frac{h_{ib} \cdot 5 \text{ k}\Omega}{5 \text{ k}\Omega + h_{ib}} \right)$$

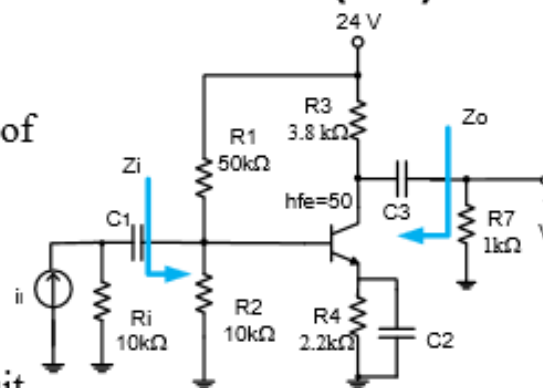
4) Output Impedance

$$Z_o \Big|_{\text{all independent sources killed (i.e. } v_i=0 \text{ or short)}} = 4 \text{ k}\Omega$$

Example: Common Emitter (CE)

- 1) From DC Analysis,
we find Q - point and value of

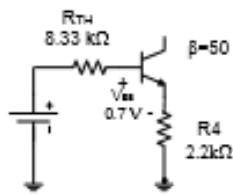
$$h_{ie} = \frac{V_T}{I_{BQ}}$$



Thevenin's equivalent circuit
as seen from the base

$$V_{TH} = \frac{10 \text{ k}\Omega}{10 \text{ k}\Omega + 50 \text{ k}\Omega} \cdot 24 \text{ V} = 4 \text{ V}$$

$$R_{TH} = 10 \text{ k}\Omega // 50 \text{ k}\Omega = 8.33 \text{ k}\Omega$$

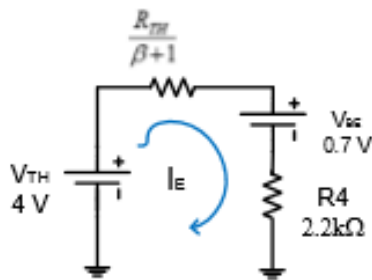


$$4 = 8.33 \text{ k}\Omega \cdot I_B + V_{BE} + 2.2 \text{ k}\Omega \cdot I_E$$

But, $I_E = (1 + \beta)I_B$

$$\text{Solve for } I_E = \frac{4 - 0.7}{\frac{8.33 \text{ k}\Omega}{(1 + 50)} + 2.2 \text{ k}\Omega} = 1.4 \text{ mA}$$

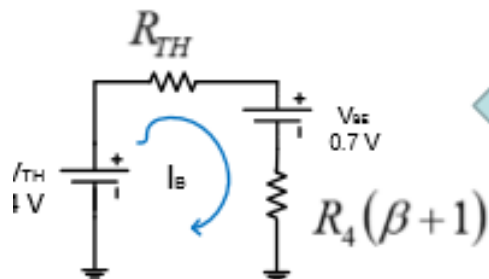
$$h_{ie} = \frac{V_T}{I_{BQ}} = \frac{25.69 \text{ mV}}{\frac{1.4 \text{ mA}}{51}} = 928 \Omega$$



Here we have base reflected to emitter

$$I_B \Rightarrow I_E = (\beta + 1)I_B$$

$$R_B \Rightarrow \frac{R_B}{\beta + 1}$$

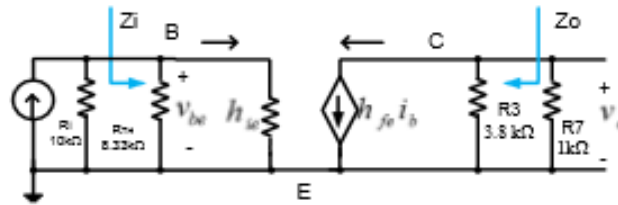


Here we have emitter reflected to base

$$I_E \Rightarrow I_B = \frac{I_E}{(\beta + 1)}$$

$$R_E \Rightarrow R_E(\beta + 1)$$

AC small signal Equivalent Circuit



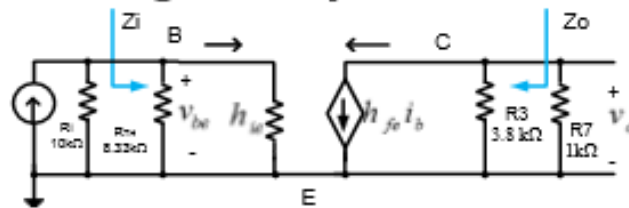
$$1) A_V = \frac{v_o}{v_i}$$

$$A_V = \frac{v_o}{v_i} = \frac{v_o}{i_b} \cdot \frac{i_b}{v_i}$$

$$v_o = -h_{fe} i_b \cdot (R_3 // R_7) \Rightarrow -h_{fe} \cdot (R_3 // R_7) \cdot \left(\frac{1}{h_{ie}} \right)$$

$$i_b = \frac{v_i}{h_{ie}} \Rightarrow -50 \cdot (3.8 \text{ k}\Omega // 1 \text{ k}\Omega) \cdot \left(\frac{1}{928 \Omega} \right) = -42.7$$

AC small signal Equivalent Circuit

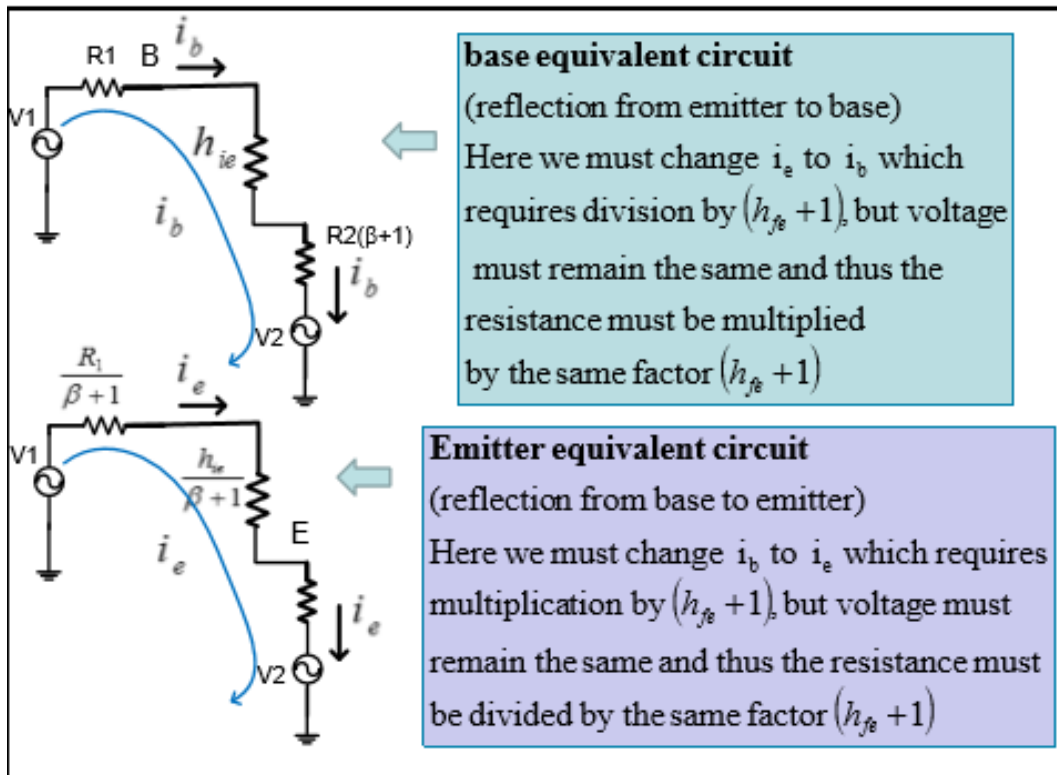


$$2) Z_i = R_{TH} // h_{ie} \\ = 8.33 \text{ k}\Omega // 928 \Omega$$

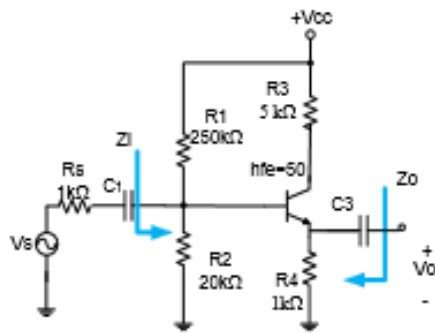
only elements to the right of arrow are considered
according to the given direction of the arrow

$$3) Z_o \Big|_{\text{all independant sources killed (i.e. } v_i=0 \text{ or short)}} = 3.8 \text{ k}\Omega$$

here $h_{fe} \cdot i_b = 0$ since $i_b = 0$ ($v_i = 0$ - killed)



Common Collector Amplifier



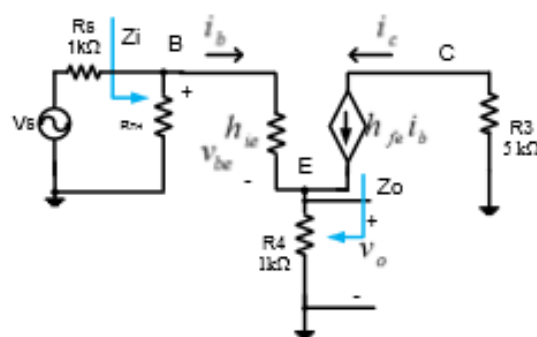
Given

$$h_{ie} = 1k\Omega$$

$$h_{fe} = \beta = 50$$

Find A_v, A_i, Z_i, Z_o

AC small signal Equivalent Circuit

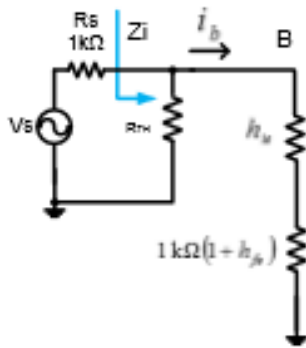


$$1) A_v = \frac{v_o}{v_s}$$

$$v_o = 1k\Omega \cdot i_e$$

$$i_e = i_b (h_{fe} + 1)$$

i_b can be found from base equivalent circuit



$$R_{TH} = 20 \text{ k}\Omega // 250 \text{ k}\Omega$$

$$i_b = i_i \frac{R_{TH}}{(R_{TH}) + (h_{ie} + 1 \text{ k}\Omega(h_{fe} + 1))}$$

$$i_i = \frac{V_S}{R_S + (R_{TH} // (h_{ie} + 1 \text{ k}\Omega(h_{fe} + 1)))}$$

$$\therefore A_v = \frac{v_o}{v_s} = \frac{v_o}{i_e} \cdot \frac{i_e}{i_b} \cdot \frac{i_b}{i_i} \cdot \frac{i_i}{v_s}$$

$$= (1 \text{ k}\Omega) \cdot (h_{fe} + 1) \left(\frac{R_{TH}}{(R_{TH}) + (h_{ie} + 1 \text{ k}\Omega(h_{fe} + 1))} \right) \left(\frac{1}{R_S + (R_{TH} // (h_{ie} + 1 \text{ k}\Omega(h_{fe} + 1)))} \right)$$

$$= 0.915 < 1$$

$$3) Z_I = (R_{TH} // (h_{ie} + 1 \text{ k}\Omega(h_{fe} + 1)))$$

$$= 13.66 \text{ k}\Omega \text{ (high)}$$

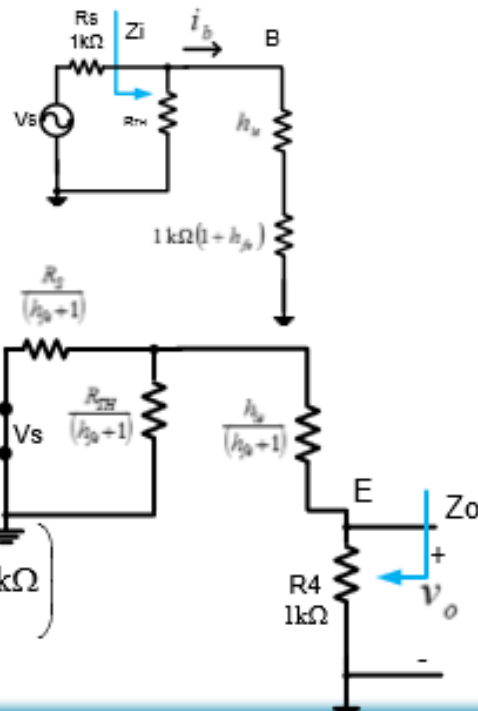
Emitter Equivalent Circuit

& $V_S = 0$

$$Z_o|_{V_S=0} = \left(\frac{(R_S // R_{TH}) + h_{ie}}{(h_{fe} + 1)} // 1 \text{ k}\Omega \right)$$

$$= \left(\left(\left(\frac{R_S}{(h_{fe} + 1)} // \frac{R_{TH}}{(h_{fe} + 1)} \right) + \frac{h_{ie}}{(h_{fe} + 1)} \right) // 1 \text{ k}\Omega \right)$$

$$= 36.8 \Omega \text{ (low)}$$



4. FET dc analysis

Example

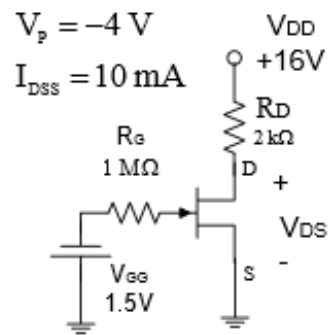
$$V_{GS} = V_G - V_S = -1.5 - 0 = -1.5 \text{ V}$$

Assuming JFET is in pinch off region

$$\begin{aligned} 1) \quad I_D &= I_{DSS} \left(1 - \frac{V_{GS}}{V_P} \right)^2 \\ &= 10 \text{ mA} \left(1 - \frac{-1.5}{-4} \right)^2 \\ &= 3.9 \text{ mA} \end{aligned}$$

$$\begin{aligned} 2) \quad V_{DS} &= V_{DD} - I_D R_D \\ &= 16 - ((2\text{k})(3.9 \text{ mA})) \\ &= 8.2 \text{ V} \end{aligned}$$

$$\begin{aligned} 3) \text{ check for } & |V_{DS}| > |V_P| - |V_{GS}| ? \\ & |8.2| > |-4| - |-1.5| \\ & \text{assumption is true} \end{aligned}$$



19

Example

V_S must be more positive than V_G

to keep the gate – source junction reverse biased

$$V_S = I_D R_S$$

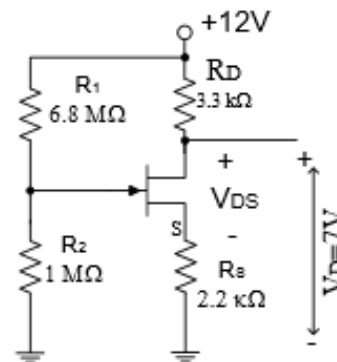
$$V_{GS} = \frac{R_2 V_{DD}}{R_1 + R_2}$$

$$V_{GS} = V_G - I_D R_S$$

$$V_{GS} = \frac{R_2 V_{DD}}{R_1 + R_2} - I_D R_S$$

$$V_D = V_{DD} - I_D R_D = 7 \text{ V}$$

$$I_D = \frac{V_{DD} - V_D}{R_D} = \frac{12 - 7}{3300} = 1.52 \text{ mA}$$



28

Example

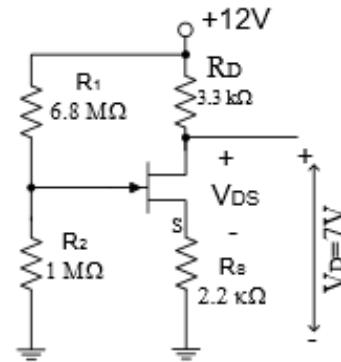
$$V_s = I_D R_s = (1.52 \text{ mA}) (2.2 \text{ k}\Omega) = 3.34 \text{ V}$$

$$V_G = \frac{1 \text{ M}}{1 \text{ M} + 6 \text{ M}} 15 = 1.54 \text{ V}$$

$$V_{GS} = 1.54 - 3.34 = -1.8 \text{ V} < 0 \quad \Leftarrow \text{OK}$$

also

$$I_D = \frac{V_s}{R_s} = \frac{3.34}{2200} = 1.52 \text{ mA}$$



Example

Suppose that the DMOSFET is in the pinch off region

$$I_{DS} = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2 \quad \dots\dots\dots 1$$

$$V_{GS} = V_G - V_S = V_G$$

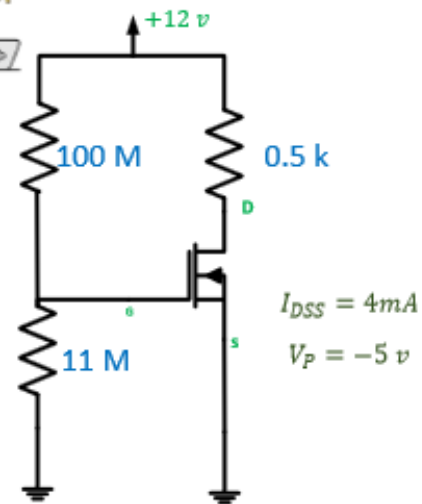
$$V_G = \frac{11 \text{ M}}{11 \text{ M} + 100 \text{ M}} (12) = 1.19 \text{ v} \quad \dots\dots\dots 2$$

sub 2 into 1, we obtain

$I_{DS} = 6.13 \text{ mA} > I_{DSS}$!! **THIS IS POSSIBLE AND DMOSFET WILL OPERATE IN ENHANCEMENT MODE**

$$V_{DS} = V_{DD} - 0.5 \text{ K} I_{DS} = 8.93 \text{ v}$$

$$V_{DS} > V_{GS} - V_P = 6.19 \text{ v}$$



$$I_{DSS} = 4 \text{ mA} \quad V_P = -5 \text{ v}$$

Example

$$I_{DS} = K_n (V_{GS} - V_T)^2 \dots\dots\dots 1$$

$$V_{GS} = V_G - V_S$$

$$V_G = \frac{22M}{22M + 47M} (18) = 5.74V$$

$$V_S = (0.5K) I_{DS}$$

$$V_{GS} = 5.74 - (0.5K) I_{DS} \dots\dots\dots 2$$

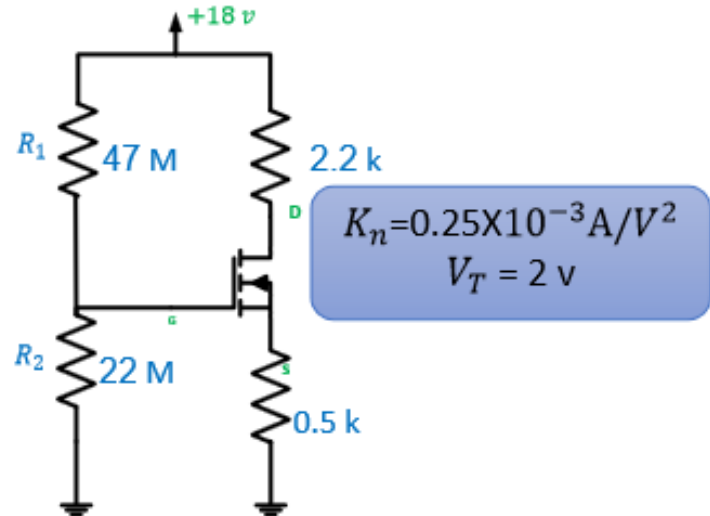
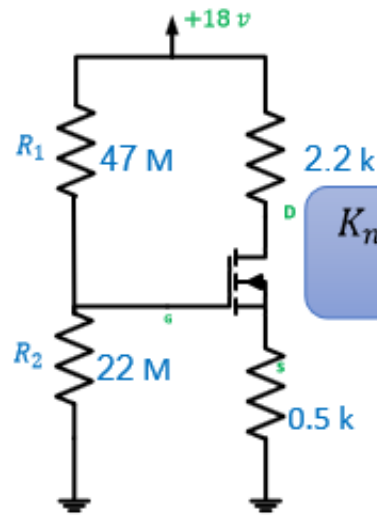
solving for V_{GS} :

$$V_{GS} = 4.78V \quad \checkmark$$

$$= -8.78V \quad \times$$

$$I_{DS} = 1.92m A$$

$$V_{DS} = 12.82 > |V_{GS} - V_T|$$



5. FET amplifier ac analysis

Definition: Transconductance g_m

For JFETs and DMOSFETs

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \frac{2I_{DSS}}{|V_P|} \left[1 - \frac{V_{GS}}{V_P} \right]$$

$$g_m = g_{m0} \left[1 - \frac{V_{GS}}{V_P} \right] = g_{m0} \sqrt{\frac{I_D}{I_{DSS}}} \quad g_{m0} = \frac{2I_{DSS}}{|V_P|}$$

For EMOSFET

$$I_D = K(V_{GS} - V_{GS(TH)})^2 \quad \Rightarrow \quad g_m = \frac{\partial I_D}{\partial V_{GS}} = 2K(V_{GS} - V_{GS(TH)})$$

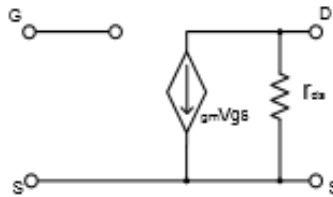
$$K = \frac{I_D}{(V_{GS} - V_{GS(TH)})^2} \quad (V_{GS} - V_{GS(TH)}) = \sqrt{\frac{I_D}{K}}$$

$$\therefore g_m = 2K \sqrt{\frac{I_D}{K}} = 2\sqrt{\frac{I_D K^2}{K}} = 2\sqrt{I_D K}$$

AC Small Signal Equivalent Circuit (MODEL Valid for all FET Types)

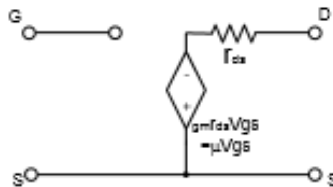
- In ac

$$g_m = \frac{i_d}{v_{gs}} \Rightarrow i_d = g_m v_{gs}$$



- Or

$$\mu = g_m r_{ds} \text{ - amplification factor}$$



Example: Phase Splitting circuit

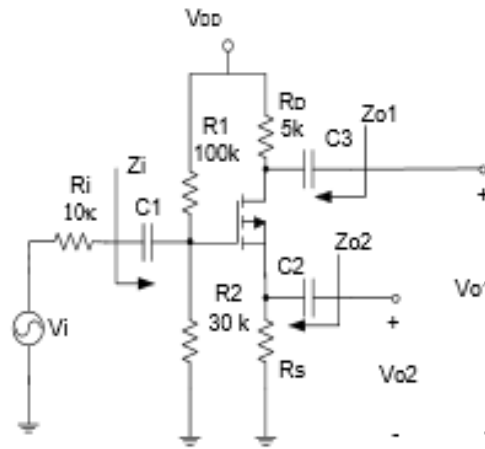
- Two outputs:

- Vo1 from drain
- Vo2 from source

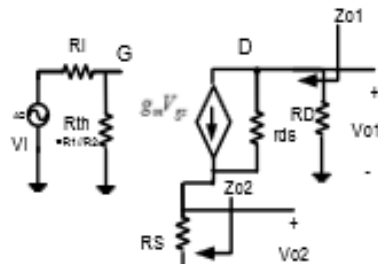
Find A_v, A_s, Z_{O1}, Z_{O2} and Z_i

$$r_{ds} = 100 \text{ k}\Omega$$

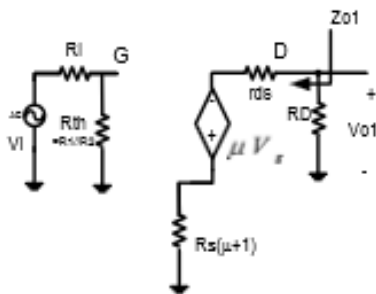
$$g_m = 1 \text{ mS}$$



Solution: ac ss equivalent circuit



1) To Find Z_{O1}, V_{O1} Drain equivalent circuit is required since both of these quantities are seen from the drain

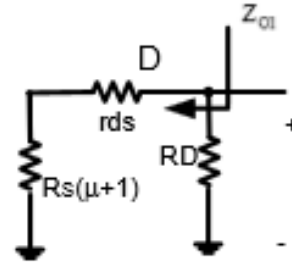
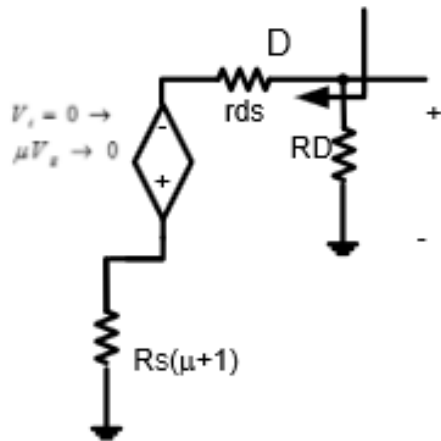


$$V_{o1} = \frac{R_D}{R_D + r_{ds} + R_s(\mu + 1)} (-\mu V_s)$$

$$V_s = V_i \frac{R_{th}}{R_{th} + R_i}$$

$$A_v = \frac{V_{o1}}{V_i} = (-\mu) \frac{R_D}{R_D + r_{ds} + R_s(\mu + 1)} \cdot \frac{R_{th}}{R_{th} + R_i}$$

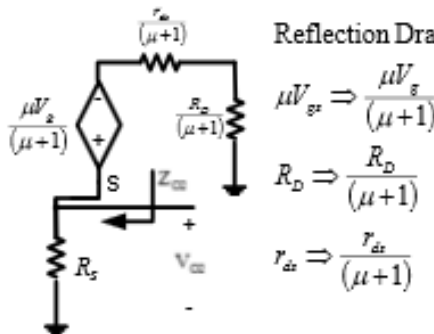
2) To Find $Z_{O1} \Big|_{V_i=0, V_g=0}$



$$Z_{O1} \Big|_{V_i=0, V_g=0} = R_D \parallel [r_{ds} + R_s(\mu + 1)]$$

Solution: continued

3) To Find Z_{O2}, V_{O2} Source equivalent circuit is required since both of these quantities are seen from the source

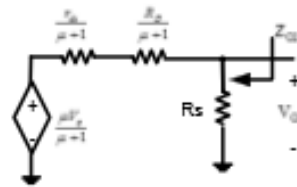


Reflection Drain to Source

$$\mu V_{gs} \Rightarrow \frac{\mu V_{\varepsilon}}{(\mu + 1)}$$

$$R_D \Rightarrow \frac{R_D}{(\mu + 1)}$$

$$r_{ds} \Rightarrow \frac{r_{ds}}{(\mu + 1)}$$



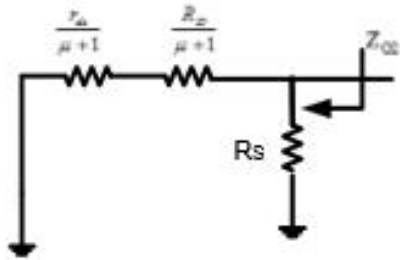
$$V_{o2} = \frac{R_s}{R_D + r_{ds} + R_s} \left(\frac{\mu V_{\varepsilon}}{(\mu + 1)} \right)$$

$$V_{\varepsilon} = V_i \frac{R_{ik}}{R_{in} + R_i}$$

$$A_{v2} = \frac{V_{o2}}{V_i} = \frac{\mu}{(\mu + 1)} \frac{R_s}{R_D + r_{ds} + R_s} \cdot \frac{R_{ik}}{R_{in} + R_i}$$

Solution: continued

4) To Find $Z_{O2} \Big|_{\substack{V_i=0 \\ V_g=0}}$



$$Z_{O2} \Big|_{\substack{V_i=0 \\ V_g=0}} = R_S \parallel \left[\frac{r_{ds} + R_D}{(\mu + 1)} \right]$$



$$Z_{O2} \Big|_{\substack{V_i=0 \\ V_g=0}} \Big|_{r_{ds} \rightarrow \infty} = R_S \parallel \frac{1}{g_m}$$

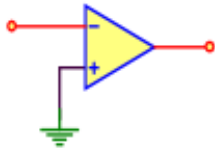
$$\text{since } \lim_{r_{ds} \rightarrow \infty} \frac{r_{ds} + R_D}{\mu + 1} = \lim_{r_{ds} \rightarrow \infty} \frac{r_{ds} + \frac{R_D}{r_{ds}}}{\frac{g_m r_{ds}}{r_{ds}} + \frac{R_D}{r_{ds}}}$$

$$= \lim_{r_{ds} \rightarrow \infty} \frac{1 + \frac{R_D}{r_{ds}}}{g_m + \frac{1}{r_{ds}}} = \frac{1}{g_m}$$

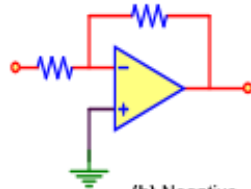
$$Z_i = R_{th} = R_1 \parallel R_2$$

6. Op-amp circuits

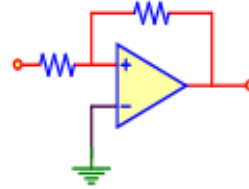
OP-AMP CONFIGURATIONS



(a) No Feedback
(open loop
comparator circuit)



(b) Negative
Feedback



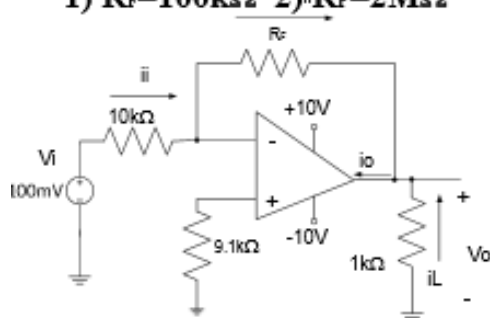
(c) Positive Feedback

- No feedback : Open loop (used in comparators)
- Negative feedback : Feedback to the inverting input (Used in amplifiers)
- Positive feedback : Feedback to the non inverting input (Used in oscillators) and Schmitt triggers (comparators with hysteresis)

Example

Find the value of V_o and I_o and verify if the opamp is in linear or saturation mode for two values of feedback resistor; assume $I_o(\text{max})=20 \text{ mA}$:

1) $R_F=100\text{k}\Omega$ 2) if $R_F=2\text{M}\Omega$



Important

$I_o(\text{max})$ is few mA for most opamps which limits the values of resistors to be used to kohm range

$$V_p = V(+)=0V$$

$$i_1 = \frac{V_1}{R_1}$$

$$i_i = i_f = \frac{V_i}{10\text{k}} = \frac{100 \text{ mV}}{10 \text{ k}\Omega} = 10 \mu\text{A}$$

$$1) V_o = -\frac{R_F}{10 \text{ k}\Omega} V_i = -10 V_i = -1V$$

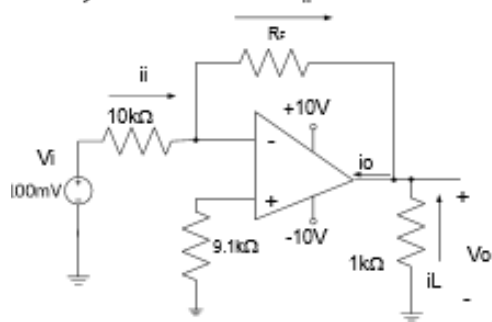
$$V_o > -V_{\text{sat}}$$

Linear mode

$$i_o = i_f + i_L$$

$$i_L = \frac{1V}{1k} = 1mA$$

$$i_o = 10\mu A + 1mA = 1.01mA < I_o(max)$$

Example continued2) $R_F = 2M\Omega$ 

$$i_i = i_f = \frac{V_i}{10k} = \frac{100mV}{10k} = 10\mu A$$

$$2) V_o = -\frac{R_F}{10k} V_i = -200V_i = -20V$$

$$V_o < -V_{sat}$$

Saturation mode $\Rightarrow V_o$ is limited to $-V_{sat}$

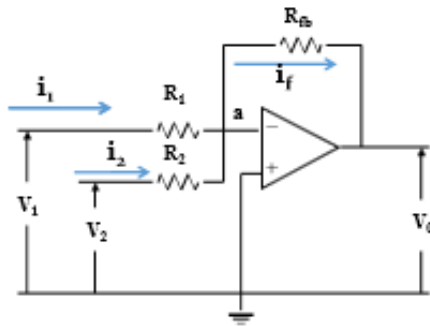
$$\therefore V_o = -V_{sat} = -8V$$

$$i_L = \frac{8V}{1k} = 8mA$$

$$i_o = 10\mu A + 8mA = 8.01mA < I_o(max)$$

Inverting Adder or Summing Amplifier

Summing Amplifier: This is an application of inverting amplifier



$$V_p = V_{(+)} = 0V$$

$$i_1 = \frac{V_1}{R_1} \quad i_2 = \frac{V_2}{R_2}$$

$$V_o = -R_f i_f$$

$$i_f = i_1 + i_2$$

$$V_o = - \left[\left(\frac{R_{fb}}{R_1} \right) V_1 + \left(\frac{R_{fb}}{R_2} \right) V_2 \right] \quad V_o = -R_f [i_1 + i_2]$$

If $R_1 = R_2 = R_{fb}$ then,

$$V_o = - [V_1 + V_2]$$

Therefore, we can add signals with an op amp

Example: Non-inverting Amplifiers

Example: Find V_o for the following op amp configuration.

$$V_x = \frac{6k}{6k + 2k} 4V$$

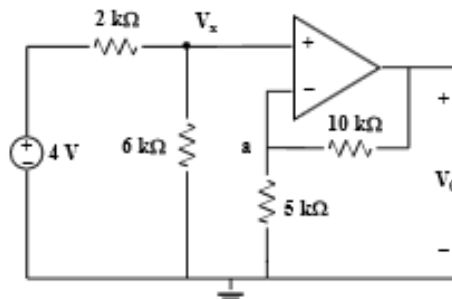
$$V_x = 3V$$

$$V_o = \left(1 + \frac{R_F}{R_i} \right) V_x$$

$$V_o = \left(1 + \frac{10k}{5k} \right) 3V$$

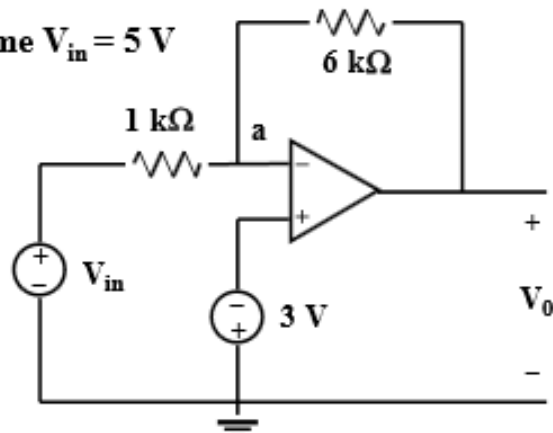
$$V_o = 9V$$

Make sure that: $-V_{sat} < V_o < +V_{sat}$



Example Consider the op amp configuration below.

Assume $V_{in} = 5\text{ V}$



$$V_o = \left(1 + \frac{6k}{1k}\right)(-3) - \left(\frac{6k}{1k}\right)(5)$$

$$= -21 - 30 = -51\text{V}$$

Since $V_o = -51\text{ V}$ (op amp will saturate and V_o will be limited to $-V_{sat}$)

Schmitt trigger.

Example: Find and sketch $V_o(t)$ and the plot of $V_o=f(V_i)$

Solution: this is a Schmitt trigger and

$$V_o = \pm V_{sat}$$

1) let $V_o = +V_{sat}$

in order for V_o to be $+V_{sat}$

$$V_d > 0$$

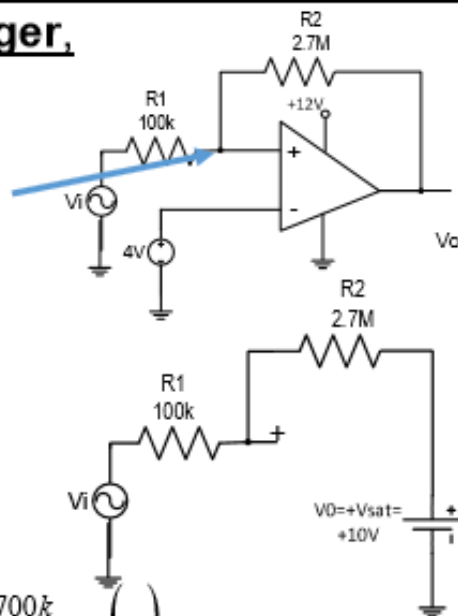
$$V_d = V(+)-V(-) > 0$$

$$V(+)>V(-)$$

$$V(-) = 4\text{V}$$

$$V(+)=\frac{100k}{(2700+100)k}(+V_{sat})+\frac{2700k}{(2700+100)k}(V_i)$$

$$\frac{100k}{(2700+100)k}(+V_{sat})+\frac{2700k}{(2700+100)k}(V_i)>4\text{V}$$



$$\frac{100k}{(2700+100)k}(10V) + \frac{2700k}{(2700+100)k}(V_i) > 4V$$

$$(V_i) > \left[4V - \left(\frac{100k}{(2700+100)k}(10V) \right) \right] \left[\frac{(2700+100)k}{2700k} \right] \implies V_i > 3.777V$$

when $V_i > 3.777 \Rightarrow V_o = +V_{sat}$; But when $V_i < 3.777 \Rightarrow V_o$ switches to $-V_{sat}$

2) let $V_o = -V_{sat} = (0 + 2) = 2V$

in order for V_o to be $-V_{sat} \implies V_d < 0$; $\therefore V(+)< V(-)$

$$V(+)= \frac{100k}{(2700+100)k}(-V_{sat}) + \frac{2700k}{(2700+100)k}(V_i)$$

$$\frac{100k}{(2700+100)k}(-V_{sat}) + \frac{2700k}{(2700+100)k}(V_i) < 4V$$

$$(V_i) < \left[4V - \left(\frac{100k}{(2700+100)k}(2V) \right) \right] \left[\frac{(2700+100)k}{2700k} \right] \implies V_i < 4.074V$$

when $V_i < 4.074V \Rightarrow V_o = -V_{sat}$; But when $V_i > 4.074 \Rightarrow V_o$ switches to $+V_{sat}$

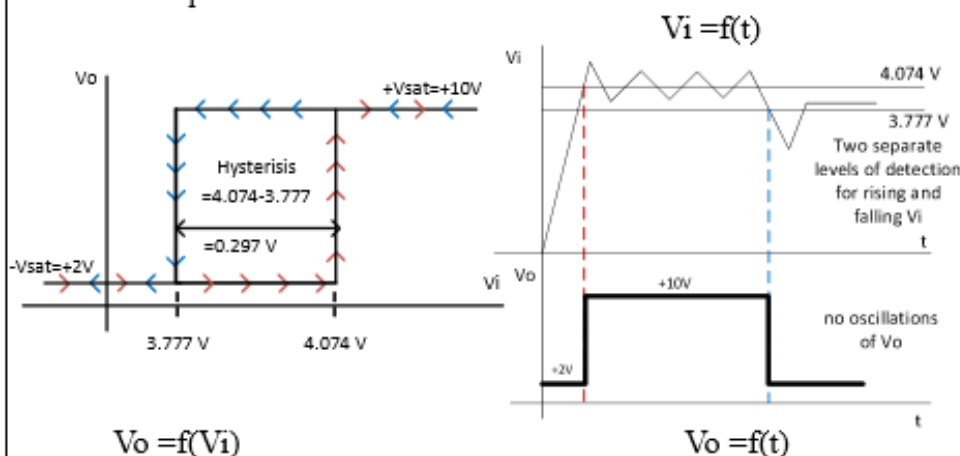
Conclusion

1) when V_i is decreasing, as long as V_i is $> 3.777V \Rightarrow V_o = +V_{sat}$;

but when V_i becomes $< 3.777 \Rightarrow V_o$ switches to $-V_{sat}$

2) when V_i is increasing, as long as V_i is $< 4.074V \Rightarrow V_o = -V_{sat}$;

but when V_i becomes $> 4.074 \Rightarrow V_o$ switches to $+V_{sat}$



Integrator

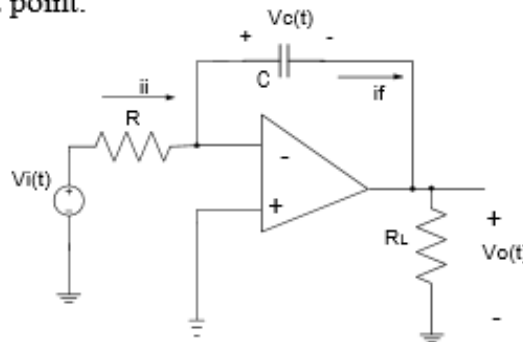
- So far, the input and feedback components have been resistors. If the feedback component used is a capacitor, the resulting connection is called an *integrator*.
- Recall that virtual ground means that we can consider the voltage at the junction of R and X_c to be ground (since $V_c = 0$ V) but that no current goes into ground at that point.

$$i_i = i_f = \frac{V_i}{R}$$

$$V_c(t) = \frac{1}{C} \int_0^t i_f(t) dt$$

$$V_o = -V_c(t)$$

$$V_o = -\frac{1}{C} \int_0^t \frac{V_i(t)}{R_i} dt = -\frac{1}{RC} \int_0^t V_i(t) dt$$



Differentiator

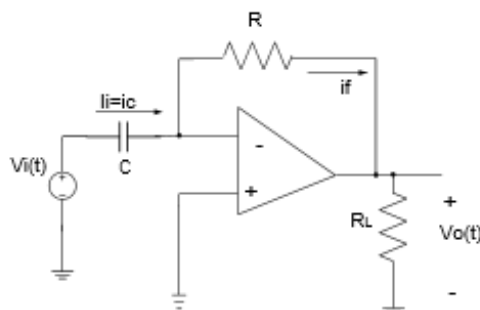
A differentiator, while not as useful as the circuit forms covered above, the differentiator does provide a useful operation, the resulting relation for the circuit being

$$i_i = i_f = i_c = C \frac{dV_i(t)}{dt}$$

$$V_c(t) = \frac{1}{C} \int_0^t i_f(t) dt$$

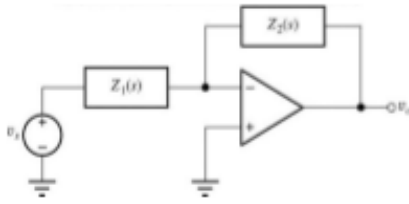
$$V_o = -i_f(t)R$$

$$V_o = -\left(C \frac{dV_i(t)}{dt} \right) (R) = -RC \frac{dV_i(t)}{dt}$$



The Active Low-pass Filter

The gain analysis of this inverting amplifier. Note $s = j\omega$.



$$A_v = \frac{\tilde{v}_o(j\omega)}{\tilde{v}_s(j\omega)} = -\frac{Z_2(j\omega)}{Z_1(j\omega)} \quad Z_1(j\omega) = R_1$$

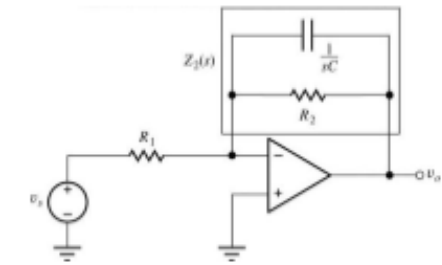
$$Z_2(j\omega) = \frac{R_2 \cdot \frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}} = \frac{R_2}{j\omega C R_2 + 1}$$

$$A_v = -\frac{R_2}{R_1} \frac{1}{(1 + j\omega C R_2)}$$

$$= \left[\frac{-K}{1 + \frac{j\omega}{\omega_c}} \right]$$

$$K = \frac{R_2}{R_1}$$

& cut-off frequency: $\omega_c = 2\pi f_c = \frac{1}{R_2 C} \quad \therefore f_c = \frac{1}{2\pi R_2 C}$

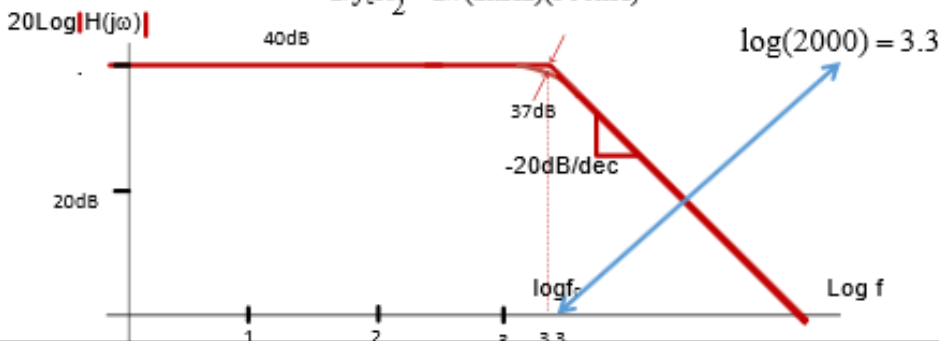


Active Low-pass Filter: Example

- **Problem:** Design an active low-pass filter
- **Given Data:** $A_v = 40 \text{ dB}$, $R_{in} = 5 \text{ k}\Omega$, $f_c = 2 \text{ kHz}$
- **Assumptions:** Ideal op amp, specified gain represents the desired low-frequency gain.
- **Analysis:** $|A_v| = 10^{40\text{dB}/20\text{dB}} = 100$
Input resistance is controlled by R_1 and voltage gain is set by R_2 / R_1 .
The cutoff frequency is then set by C.

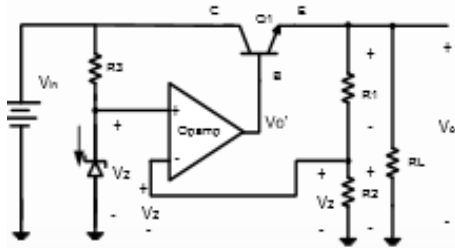
$$R_1 = R_{in} = 5 \text{ k}\Omega \quad \text{and} \quad |A_v| = \frac{R_2}{R_1} \Rightarrow R_2 = 100R_1 = 500 \text{ k}\Omega$$

$$C = \frac{1}{2\pi f_c R_2} = \frac{1}{2\pi (2 \text{ kHz})(500 \text{ k}\Omega)} = 159 \text{ pF}$$



7. Voltage regulator circuits

Opamp Series VR



Resistors R_1, R_2 are for sampling of V_o

(the current through these resistors must be small)

$$V_o = V_{R1} + V_{R2}$$

$$I_{R1} = I_{R2} = I$$

$$V_{R2} = V_Z$$

$$I = \frac{V_Z}{R_2}$$

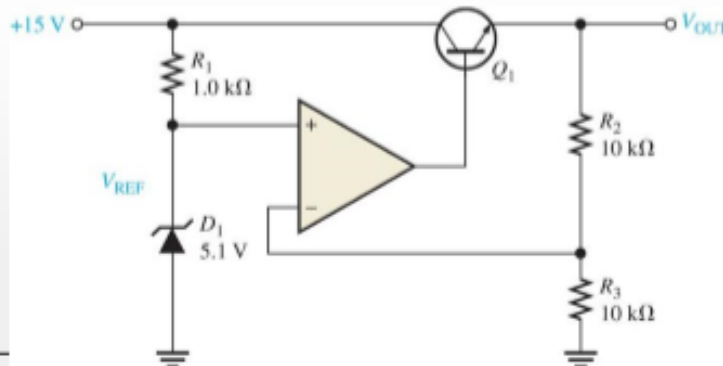
$$V_Z = V_o \frac{R_2}{R_1 + R_2}$$

$$V_o = V_Z \left(1 + \frac{R_1}{R_2} \right)$$

Example

- Determine the output voltage for the regulator below.

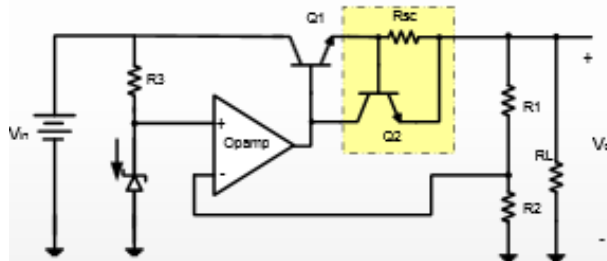
$$V_o = \left(1 + \frac{R_2}{R_3} \right) V_Z \quad \Rightarrow \quad V_o = \left(1 + \frac{10\text{k}}{10\text{k}} \right) 5.1 = 10.2 \text{ V}$$



Opamp Series VR with current limit

Current Limiting Circuit

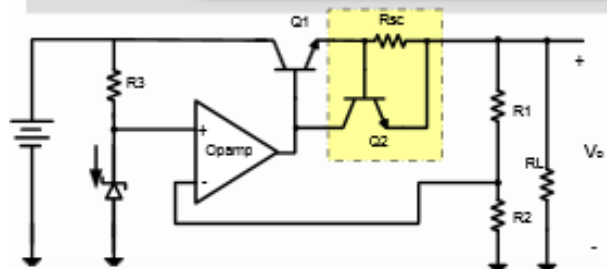
In order to protect the transistor from damage when a very high current passes through it due to a short circuit or excessive current demand at the load



1) In normal operation Q2 is off since $V_{be2} = V_{R_{sc}} < 0.7 \text{ V}$

$$2) R_{sc} = \frac{V_{be}}{I_{L(\text{Max})}} = \frac{0.7 \text{ V}}{I_{L(\text{Max})}}$$

Opamp Series with current limit



3) When $I > I_{L(\text{Max})}$,
Q2 conducts since
 $V_{be2} = V_{R_{sc}} \cong 0.7 \text{ V}$

4) Some of I_{B1} is diverted through Q2 (I_{C2})

I_{B1} is reduced so that I_L is limited to a maximum value

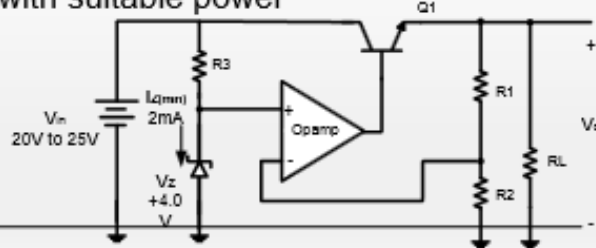
$$\text{calculated as : } I_{L(\text{Max})} = \frac{V_{be}}{R_{sc}} = \frac{0.7 \text{ V}}{R_{sc}}$$

5) Since V_{be2} cannot exceed 0.7 V , $V_{R_{sc}}$ is limited

6) This is constant current limiting

Voltage Regulators example

- Given the following series voltage regulator
- 1) Complete the design of the following voltage regulator (Find R_1 , R_2 and R_3) assuming that the voltage across the load resistor R_L is equal to 12V. Assume $I_{Z(\min)} = 2\text{mA}$.
- 2) Show how to modify the circuit to limit the load current to 1A.
- 3) Find the output voltage for the modified circuit of part 2) when the load resistor $R_L = 100\Omega$ and when $R_L = 8\Omega$.
- 4) Choose a transistor with suitable power rating



Example Continued

• SOLUTION

$$1) R_3 \leq \frac{V_{IN(\text{Min})} - V_Z}{I_{Z(\text{Min})}}$$

$$R_3 \leq \frac{20 - 4}{2 \text{ mA}} = 8 \text{ k}\Omega \text{ in order to make sure } I_Z > I_{Z(\text{Min})}$$

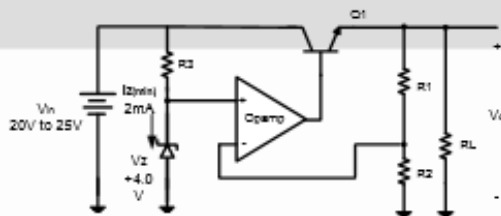
If $I_{Z(\text{max})}$ was known, then lower limit for R_3 can also be found

$$V_o = \left(1 + \frac{R_1}{R_2}\right) V_Z = 12 \text{ V}$$

$$\therefore \frac{R_1}{R_2} = \frac{V_o}{V_Z} - 1 = \frac{12}{4} - 1 = 2$$

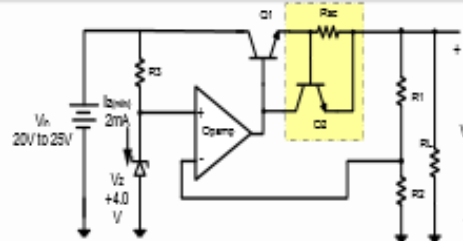


choose $R_1 = 20 \text{ k}\Omega$
 $\therefore R_2 = 10 \text{ k}\Omega$



Voltage Regulators

• SOLUTION

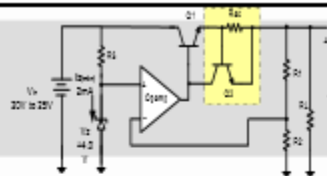


2) — The change for current limit is done by adding Q2 and Rsc as shown

$$\& R_{sc} = \frac{V_{be}}{I_{L(\text{Max})}} = \frac{0.7 \text{ V}}{1 \text{ A}} = 0.7 \Omega$$

Ex. Continued

• SOLUTION



For $R_L = 100 \text{ ohm}$, $V_o = 12 \text{ V}$, then $I_L = \frac{12 \text{ V}}{100 \Omega} = 0.12 \text{ A}$

which is smaller than $I_{L(\text{max})}$,

$\therefore V_o = 12 \text{ V}$ and is not affected by the current limit circuit

For $R_L = 8 \text{ ohm}$, $V_o = 12 \text{ V}$, then $I_L = \frac{12 \text{ V}}{8 \Omega} = 1.5 \text{ A}$

which is bigger than $I_{L(\text{max})}$, and the current limit circuit

limits the current to the maximum allowable value which is 1 A

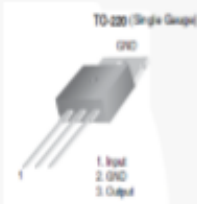
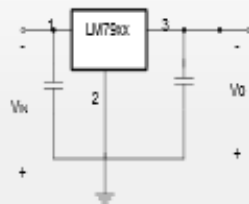
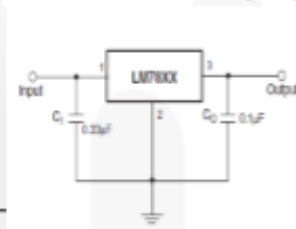
$\therefore V_o = I_{L(\text{Max})} * R_L = 1 \text{ A} * 8 \Omega = 8 \text{ V}$

3 Terminal IC Voltage Regulators

- Fixed output voltage type
- Two families exist:
 - Fixed positive output (78xx) , where xx defines the value of output voltage such as 5, 6, 8,9,12 ... etc
 - Fixed negative output (79xx) , where xx defines the value of output voltage such as -5, -6, -8,-9,-12 ... etc

LM78XX / LM78XXA

3-Terminal 1 A Positive Voltage Regulator



Fixed Voltage Regulator

Positive-Voltage Regulators in the 78XX Series

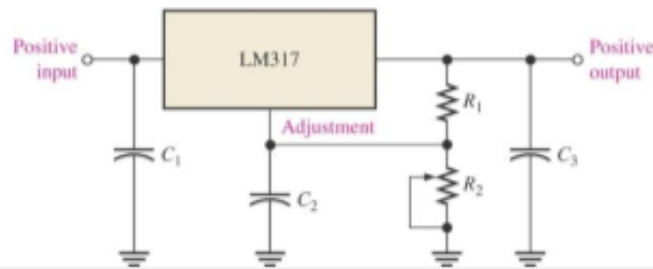
IC Part	Output Voltage (V)	Minimum V_i (V)
7805	+5	+7.3
7806	+6	+8.3
7808	+8	+10.5
7810	+10	+12.5
7812	+12	+14.5
7815	+15	+17.7
7818	+18	+21.0
7824	+24	+27.1

V_{in} must be higher than V_o by at least 2V for proper operation of the voltage regulator

Adjustable-Voltage Regulator

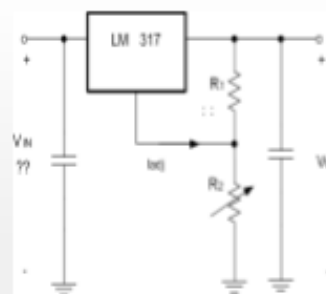
Adjustable-Voltage Regulator

- Voltage regulators are also available in circuit configurations that allow to set the output voltage to a desired regulated value.
- The LM317 is an example of an adjustable-voltage regulator, can be operated over the range of voltage from 1.25 to 35 V



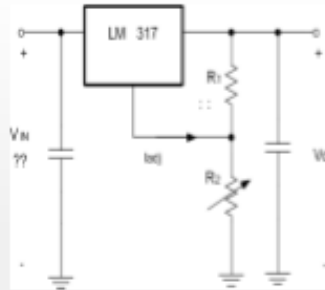
Voltage Regulators

- $I_{adj} \sim 50 \mu\text{A}$ (constant From data sheet)
- $V_{REF} = 1.25$ (always true for the 317 family)
- $V_o \sim 1.25 - 35\text{V}$
- V_o is defined by proper choice of R_1 & R_2
- $V_o = V_{R1} + V_{R2}$
- $V_{R1} = V_{REF} = I_{R1} * R_1$
- $I_{R1} = I_{REF} = V_{REF} / R_1$
- $V_{R2} = (I_{REF} + I_{ADJ}) * R_2$
- $V_o = I_{REF} * (R_1 + R_2) + I_{adj} * R_2$



Example

- Given $R_1=220\ \Omega$; $R_2=5k\Omega$ potentiometer
- $I_{adj} \approx 50\ \mu A$ (constant From data sheet)
- Find $V_o(\min)$ and $V_o(\max)$
- Find range of V_{in} ?



Voltage Regulators

$$I_{REF} = \frac{V_{REF}}{R_1} = \frac{1.25}{220\ \Omega}$$

$$V_O = I_{REF}(R_1 + R_2) + I_{adj}(R_2)$$

$$V_{O(MAX)} |_{R_2=5k\Omega} = (26.66 + 0.25) = 29.91\ V$$

$$V_{O(MIN)} |_{R_2=0k\Omega} = V_{REF} = 1.25\ V$$

The input voltage must be higher than the output by at least 2 V

$$V_{IN(MIN)} \cong 1.25 + 2 = 3.25\ V$$

$$V_{IN(MAX)} \cong 29.91 + 2 = 31.91\ V$$